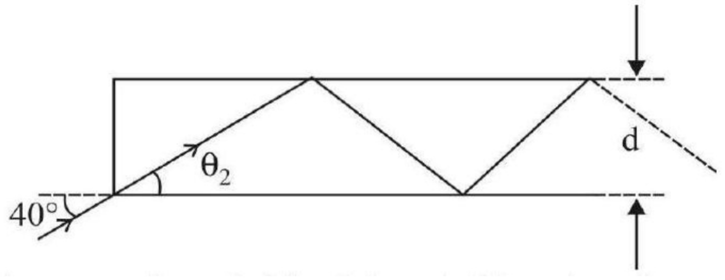
Ray Optics and Optical Instruments

1. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance (in cm) from the refracting surface at which a parallel beam of light will come to focus.

2. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field (in V/m) for the wave propogating in the glass medium will be:

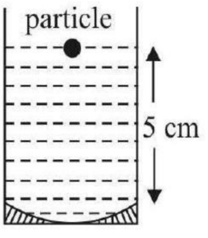
3. In figure, the optical fiber is $l = 2$ m long and has a diameter of $d = 20\mu$ m. If a ray of light is incident on one end of the fiber at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is : (refractive index of fiber is 1.31 and sin $40° = 0.64$)



4. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance (in cm) of the object for which this concave mirror, by itself would produce a virtual image would be :

5. A concave mirror for face viewing has focal length of 0.4 m. The distance (in metre) at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:

6. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d (in cm) is :



(Refractive index of water = 1.33)

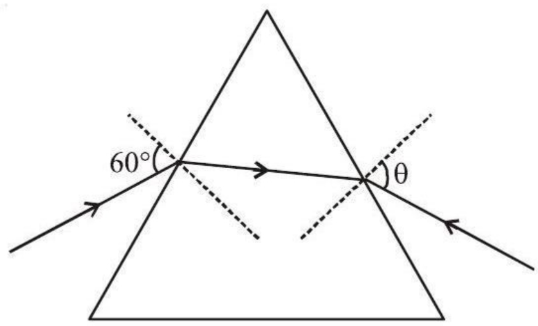
7. A simple telescope, consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm is focussed on a distant object is such a way that parallel rays comes out from the eye lens. If the object subtends an angle 2° at the objective, the angular width (in degree) of the image is

8. A ray of light falls on a glass plate of refractive index $\mu = 1.5$. What is the angle of incidence (in degree) of the ray if the angle between the reflected and refracted rays is 90° ?

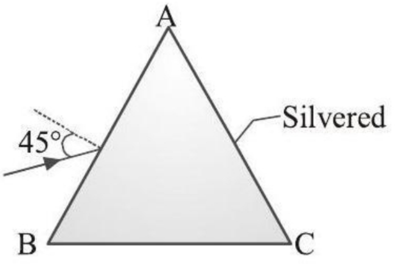
9. A coin lies on the bottom of a vessel filled with water to a depth of 40 cm. At what height (in cm) should a small electric lamp be placed above the water surface so that its image produced by the rays reflected from the water surface coincides with the image of the coin formed by the refracted rays?

10. A light ray is incident at an angle 45° with the normal to a $\sqrt{2}$ cm thick plate ($\mu = 2$). Find the shift in the path (in cm) of the light as it emerges out from the plate.

11. The monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is

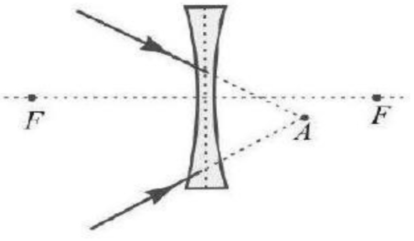


12. A prism ABC of angle 30° has its face AC silvered. A ray of light incident at an angle of 45° at the face AB retraces its path after refraction at face AB and reflection at face AC. The refractive index of the material of the prism is



13. A glass sphere of radius 5 cm has a small bubble 2 cm from its centre. The bubble is viewed along a diameter of the sphere from the side on which it lies. How far (in cm) from the surface will it appear. Refractive index of glass is 1.5.

14. A converging beam of rays is incident on a diverging lens. Having passed through the lens the rays intersect at a point 15 cm from the lens. If the lens is removed the point where the rays meet will move 5 cm closer to the mounting that holds the lens. Find focal length (in cm) of the lens.

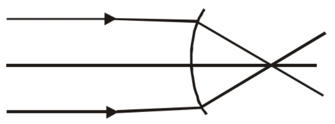


15. The magnifying power of a microscope with an objective of 5 mm focal length is 40. The length of its tube is 20 cm. Then the focal length (in cm) of the eye-piece is

SOLUTIONS

1. **(3.1)** Using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$R=7.8\,mm$



$\mu_1=1\ \mu_2=1.34$

$\Rightarrow \frac{1.34}{V}-\frac{1}{\infty}=\frac{1.34-1}{7.8}[\because u=\infty]$

$\therefore V=30.7\,mm=3.07\,cm\simeq 3.1\,cm$

2. **(24)** As 4% of light gets reflected, so only $(100-4=96\%)$ of light comes after refraction so,

$$P_{refracted}=\frac{96}{100}P_I$$

$$\Rightarrow K_2A_t^2=\frac{96}{100}K_1A_i^2$$

$$\Rightarrow r_2A_t^2=\frac{96}{100}r_1A_i^2$$

$$\Rightarrow A_t^2=\frac{96}{100}\times\frac{1}{\frac{3}{2}}\times(30)^2$$

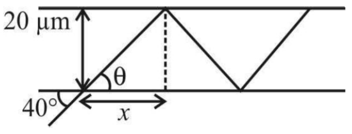
$$A_t\sqrt{\frac{64}{100}\times(30)^2}=24$$

3. **(57000)** Using Snell's law of refraction,

$1\times\sin 40°=1.31\sin\theta$

$\Rightarrow \sin\theta=\frac{0.64}{1.31}=0.49\approx 0.5$

$\Rightarrow \theta=30°$

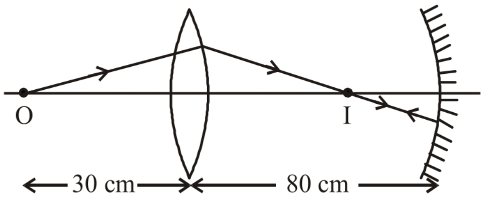


$x=20\,\mu m\times\cot\theta$

$\therefore$ Number of reflections $=\frac{2}{20\times10^{-6}\times\cot\theta}$

$$=\frac{2\times10^6}{20\times\sqrt{3}}=57735\approx 57000$$

4. (10) For lens



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

or $\frac{1}{v}-\frac{1}{-30}=\frac{1}{20}$

$\therefore$ $v = +60$ cm

According to the condition, image formed by lens should be the centre of curvature of the mirror, and so

$2f' = 20$ or $f' = 10$ cm

5. (0.32) $+5=-\frac{v}{u}\Rightarrow v=-5u$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\Rightarrow$ $\frac{1}{-5u}+\frac{1}{u}=\frac{-1}{0.4}$

$\therefore$ u = – 0.32 m.

6. (8.8) If v is the distance of image formed by mirror, then

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

or $\frac{1}{v}+\frac{1}{-5}=\frac{1}{-20}$

$\therefore$ $v=\frac{20}{3}$ cm

Distance of this image from water surface

$=\frac{20}{3}+5=\frac{35}{3}$ cm

Using, $\frac{RD}{AD}=\mu$

$\therefore$ $AD = d = \frac{RD}{\mu}=\frac{(35/3)}{1.33}=8.8$ cm

7. (24) $f_0 = 60$ cm and $f_e = 5$ cm

Magnification $M = \frac{f_0}{f_e}=\frac{\beta}{\alpha}$

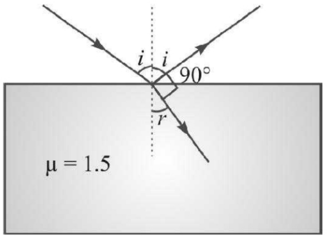
or $\frac{60}{5} = \frac{\beta}{2°}$

$\therefore$ $\beta = 24°$

8. **(57)** If i and r are the angle of incident and angle of refraction respectively, then

$$i + r = 90°$$

$$\therefore \quad r = 90° - i$$



By Snell's law, $\dfrac{\sin i}{\sin r} = \mu$

or $\dfrac{\sin i}{\sin(90° - i)} = \mu$

or $\dfrac{\sin i}{\cos i} = \mu$

or $\tan i = \mu$

$\therefore \quad i = \tan^{-1}(\mu) = \tan^{-1}(1.5)$

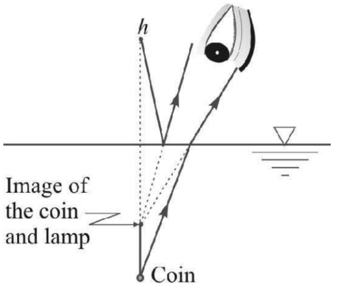
$\simeq 57°$

9. **(30)** If y is the apparent distance of the coin, then

$$y = \frac{40}{\mu}$$

$$= \frac{40}{4/3} = 30\,\text{cm}$$

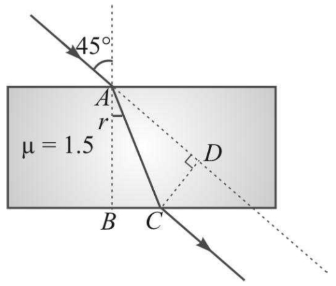
If h is the height of the lamp from the water surface, then its image will be at a distance h below the water surface. Both the images to be coincide, $h = 30$ cm



10. **(0.62)** By Snell's law

$$\frac{\sin i}{\sin r} = \mu$$

$$\therefore \quad \sin r = \frac{\sin i}{\mu} \quad \ldots(i)$$



In Δ ABC, $AC = \frac{AB}{\cos r}$

In Δ ACD, $CD = AC\sin(i-r)$

$$= \frac{AB}{\cos r}\sin(i-r) \quad \ldots(ii)$$

On substituting $AB = \sqrt{2}$ cm, $i = 45°$

and solving $CD = 0.62$ cm.

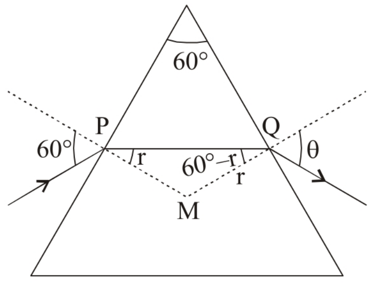
11. **(2)** Here $\angle MPQ + \angle MQP = 60°$. If $\angle MPQ = r$ then $\angle MQP = 60 - r$

Applying Snell's law at P

$\sin 60° = n \sin r$ ...(i)

Differentiating w.r.t 'n' we get

$$O = \sin r + n\cos r \times \frac{dr}{dn} \quad \text{...(ii)}$$



Applying Snell's law at Q

$\sin\theta = n\sin(60° - r)$ ...(iii)

Differentiating the above equation w.r.t 'n' we get

$$\cos\theta\frac{d\theta}{dn} = \sin(60°-r) + n\cos(60°-r)\left[-\frac{dr}{dn}\right]$$

$$\therefore \cos\theta\frac{d\theta}{dn} = \sin(60°-r) - n\cos(60°-r)$$

$$\left[-\frac{\tan r}{n}\right] \quad \text{[from (ii)]}$$

$$\therefore \quad \frac{d\theta}{dn} = \frac{1}{\cos\theta}\left[\sin(60°-r) + \cos(60°-r)\tan r\right] \text{...(iv)}$$

From eq. (i), substituting $n=\sqrt{3}$ we get $r=30°$

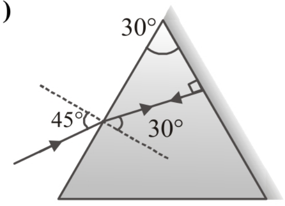
From eq (iii), substituting $n=\sqrt{3}$, $r=30°$

we get $\theta=60°$

On substituting the values of r and θ in eq (iv) we get

$$\frac{d\theta}{dn}=\frac{1}{\cos 60°}[\sin 30°+\cos 30° \tan 30°]=2$$

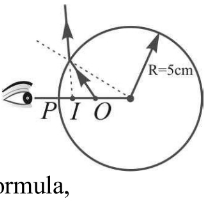
12. ($\sqrt{2}$)



$$\mu = \frac{\sin 45°}{\sin 30°} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}.$$

13. **(2.5)** For the object O,

$$u = -(PO) = -3 \text{ cm}$$



By refraction formula,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}, \text{ we have}$$

$\mu_1=1.5$, $\mu_2=1$, and $R=-5$cm

$$\therefore \quad \frac{1}{v}-\frac{1.5}{-3} = \frac{1-1.5}{-5}$$

$$\Rightarrow \quad v = -2.5 \text{ cm}$$

14. **(30)** For concave lens, $u = +10$ cm (virtual object)

and $v = +15$ cm

We have $$\frac{1}{+15}-\frac{1}{+10} = \frac{1}{f}$$

$$\therefore \quad f = -30 \text{ cm.}$$

15. **(2.5)** $$M = -\frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$$

or $$-40 = -\frac{20}{5}\left(1+\frac{25}{f_e}\right)$$

or $$f_e = 2.5 \text{ cm}$$